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CHARACTERIZATION OF ORTHOGONAL 24-RUN 2^6 FACTORIAL DESIGNS DERIVABLE FROM SATURATED TWO-SYMBOL ORTHOGONAL ARRAYS OF STRENGTH 2, SIZE 24, 23 CONSTRAINTS AND INDEX 6

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Abstract. This paper is concerned with two-symbol orthogonal arrays of strength 2, size 24, 6 constraints and index 6 which are obtained by selecting 6 columns from saturated two-symbol orthogonal arrays having maximal constraints. In [15], it has been shown that there are precisely 1,317 isomorphic classes of such arrays with respect to the permutation of factors and levels within factors. In this paper, the statistical properties of those orthogonal 24-run 2^6-factorial designs which are associated with the above are investigated.

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§0. Introduction and summary

Orthogonal 2^m factorial designs have widely been used in factor screening and related experiments. Among others, such designs obtained by assigning factors to the appropriate columns of a saturated orthogonal array have been recommended for practical use (see, e.g., Taguchi [6,7], and Box and Hunter [1,2]). All saturated orthogonal arrays of size 4λ are isomorphic to each other with respect to the permutation of $4\lambda - 1$ columns (factors) and symbols (levels) in each case of $\lambda = 1, 2$ and 3, respectively. However, it has been shown in Yamamoto, Fujii, Hyodo and Yumiba [8,9,12]), there are 5, 3 and 130 isomorphic classes of saturated orthogonal arrays of size 4λ in those cases of $\lambda = 4, 5$ and 6, respectively. The possibility, therefore, of obtaining so many useful orthogonal 2^m factorial designs from such saturated orthogonal arrays is expected as have been illustrated in Yamamoto, Fujii, Hyodo and Yumiba [10].

The classification of all orthogonal 2^5 and 2^6 factorial designs having 16 and 20 runs derivable from the representative saturated orthogonal arrays of size 16 and 20 has been done in Yamamoto, Fujii, Hyodo and Yumiba [11]. Representative designs of those isomorphic classes and their characteristic vectors have been given there. The results of the classification of all orthogonal 24-run 2^5 factorial designs derivable from two-symbol orthogonal arrays of size 24, strength 2, 23 (maximal) constraints and index 6 have been given in Yamamoto, Fujii, Hyodo and Yumiba [13]. The number of those isomorphic classes is 63 and it is just the same with that of all orthogonal arrays of size 24, 5 constraints and index 6 given in Namikawa, Fujii and Yamamoto [4]. The results imply the classification of all orthogonal 24-run 2^5 factorial designs. Statistical properties of those 63 representative designs with a special reference to the A-optimal one are included.

Recently, all orthogonal arrays having 6 constraints, which are derivable from representative 130 two-symbol orthogonal arrays of size 24, strength 2, 23 (maximal) constraints and index 6, i.e., 2-OA(2,23,6), have been classified computationally (Yumiba, Hyodo and Yamamoto [15]). The number of isomorphic classes of corresponding orthogonal 24-run 2^6 factorial designs amounts to 1,317.

In this paper, a partial list including significant designs among 1,317 representatives is given in Table 1. The complete list of those results can be seen in Hyodo, Yumiba and Yamamoto [3]. A subset of 130 saturated orthogonal arrays which is sufficient to give all of those representative designs can be seen in Appendix of Yamamoto, Fujii, Hyodo and Yumiba [12]. The total sum of the variances of the estimates up to two-factor interactions as well as the partial sum of the variances of the estimates of main effects and that of two-factor interactions are included in Table 1, provided the design is of resolution V. Only the total sum of the variances of the estimates of main effects is included in this Table 1 if the design is of resolution IV. In order to apply the elementary transformation algorithm to the normal equation for obtaining exactly the rank of information matrix and other characteristics given in Table 1, a numerical routine for the integer of arbitrary length has been prepared. This routine makes us possible to obtain the results summarized above. Specific features of some of those representative designs of interest are also given in this paper.

§1. Preliminaries

Consider a 2^m factorial experiment with m factors, $F(1), F(2), \dots,$ and $F(m)$, each at two levels 0 and 1. Let $\theta\{\phi\}; \theta\{p\}$; and, in general, $\theta\{K\} K = \{p_1, p_2, \dots, p_k\} \subset \Omega = \{1, 2, \dots, m\}$, be various factorial effects called the general mean; the main effect of the factor $F(p)$; and the k-factor interaction of $k(2 \le k \le m)$ factors $F(p_1), F(p_2), \dots,$ and $F(p_k)$, respectively.

Let T be a fraction of 2^m factorial design of m factors composed of n binary assemblies $(j_1^{(\alpha)}, j_2^{(\alpha)}, \dots, j_m^{(\alpha)})$ with $j_p^{(\alpha)} = 1$ or 0 for $p = 1, 2, \dots, m$ and $\alpha = 1, 2, \dots, n$, and suppose $\boldsymbol{y}(T)$ be the corresponding vector of observations, i.e.,

$$(1.1) \qquad T = \begin{bmatrix} j_1^{(1)}, j_2^{(1)}, \dots, j_m^{(1)} \\ \vdots \\ j_1^{(\alpha)}, j_2^{(\alpha)}, \dots, j_m^{(\alpha)} \\ \vdots \\ j_1^{(n)}, j_2^{(n)}, \dots, j_m^{(n)} \end{bmatrix}, \quad \boldsymbol{y}(T) = \begin{bmatrix} y(j_1^{(1)}, j_2^{(1)}, \dots, j_m^{(1)}) \\ \vdots \\ y(j_1^{(\alpha)}, j_2^{(\alpha)}, \dots, j_m^{(\alpha)}) \\ \vdots \\ y(j_1^{(n)}, j_2^{(n)}, \dots, j_m^{(n)}) \end{bmatrix}$$

The vector of observations of this design T is expressed as

$$(1.2) \qquad \boldsymbol{y}(T) = E(T)\boldsymbol{\Theta} + \boldsymbol{e}(T)$$

in terms of $E(T), \boldsymbol{\Theta}$, and $\boldsymbol{e}(T)$, where $E(T)$ is the design matrix whose $(\alpha, \theta\{K\})$ element lying in the row corresponding to αth observation and the column corresponding to the factorial effect $\theta\{K\}$ is given by $\prod_{p \in K} d(j_p^{(\alpha)})$, $\boldsymbol{\Theta}$ is the column vector of factorial effects, i.e.,

$$\begin{aligned} (1.3) \quad \boldsymbol{\Theta}^t \;=\; & (\theta\{\phi\}; \theta\{1\}, \dots, \theta\{m\}; \theta\{1,2\}, \dots, \theta\{m-1, m\}; \\ & \dots; \theta\{1, 2, \dots, k\}, \dots, \theta\{m-k+1, m-k+2, \dots, m\}; \\ & \dots; \theta\{1, 2, \dots, m\}), \end{aligned}$$

and $\boldsymbol{e}(T)$ is the error vector with usual assumption that the components are distributed independently with $(0, \sigma^2)$. Here, $d(j) = -1$ or 1 according as $j = 0$ or 1 (see e.g., Yamamoto, Shirakura and Kuwada [14]).

The expectation of the αth observation of $\boldsymbol{y}(T)$ is expressed as:

$$(1.4) \qquad \eta(j_1^{(\alpha)}, j_2^{(\alpha)}, \dots, j_m^{(\alpha)}) = \sum_{u=0}^{m} \sum_{U \in \Omega(u)} \prod_{p \in U} d(j_p^{(\alpha)}) \theta\{U\},$$

where $\Omega(u)$ denotes the collection of all subsets of $\Omega = \Omega(m)$ having the cardinality u each. In particular, $\Omega(0) = \phi$.

The column vector $\boldsymbol{d}(K)$ of the design matrix $E(T)$ corresponding to the factorial effect $\theta\{K\}$ is expressed as:

$$(1.5) \qquad \boldsymbol{d}(K)^t = (\prod_{p \in K} d(j_p^{(1)}), \dots, \prod_{p \in K} d(j_p^{(\alpha)}), \dots, \prod_{p \in K} d(j_p^{(n)})).$$

In particular, $\boldsymbol{d}(\phi) = \boldsymbol{j}^t = (1, 1, \dots, 1)$ for the general mean $\theta\{\phi\}$, and $\boldsymbol{d}(p)^t = (d(j_p^{(1)}), \dots, d(j_p^{(\alpha)}), \dots, d(j_p^{(n)}))$ for the main effect $\theta\{p\}$.

Definition 1.1. (Yamamoto, Fujii, Hyodo and Yumiba [11,13]) A column vector $\boldsymbol{d}(K)$ of the design matrix $E(T)$ is called the loading vector of a factorial effect $\theta\{K\}$.

The loading vectors satisfy the following:

$$\boldsymbol{d}(U) * \boldsymbol{d}(V) = \boldsymbol{d}(U \triangle V), \tag{1.6}$$

where $\boldsymbol{x} * \boldsymbol{y}$ denotes the so-called Schur product of two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ and $U \triangle V$ denotes the symmetric difference of two subsets U and V of Ω.

Let $\|\boldsymbol{x}\|$ be a kind of magnitude called spur of the vector $\boldsymbol{x}$ being defined by the sum of its elements.

Definition 1.2. (Yamamoto, Fujii, Hyodo and Yumiba [11,13]) The spur of a loading vector $\boldsymbol{d}(K)$ is called the loading coefficient of the factorial effect $\theta\{K\}$ to the general mean $\theta\{\phi\}$ and is denoted by $\gamma(K)$.

The normal equation for estimating $\boldsymbol{\Theta}$ is given by

$$M(T)\boldsymbol{\Theta} = E(T)^t \boldsymbol{y}(T), \tag{1.7}$$

where $M(T) = E(T)^t E(T)$ is the information matrix of the design T.

Under an a priori assumption that $\ell + 1 (1 \leq \ell \leq m)$ or more higher order interactions can be assumed to be zero, the observation vector $\boldsymbol{y}(T)$ can be expressed as

$$\boldsymbol{y}(T) = E(\ell, T)\boldsymbol{\Theta}(\ell) + \boldsymbol{e}(T) \tag{1.8}$$

in terms of $E(\ell, T)$, $\boldsymbol{\Theta}(\ell)$, and $\boldsymbol{e}(T)$, where $E(\ell, T)$ is a restricted design matrix composed of those loading vectors of T corresponding up to ℓ-factor interactions, $\boldsymbol{\Theta}(\ell)$ is the vector of factorial effects up to ℓ-factor interactions, and $\boldsymbol{e}(T)$ is the error vector.

In such a situation, the normal equation for estimating $\boldsymbol{\Theta}(\ell)$ is given by

$$M(\ell, T)\boldsymbol{\Theta}(\ell) = E(\ell, T)^t \boldsymbol{y}(T), \tag{1.9}$$

where $M(\ell, T) = E(\ell, T)^t E(\ell, T)$ is the restricted information matrix of the design T.

The $(\theta\{U\}, \theta\{V\})$ element $\varepsilon(U, V)$ of the information matrix lying in the row corresponding to $\theta\{U\}$ and the column corresponding to $\theta\{V\}$, respectively, is given by

$$\varepsilon(U, V) = \|\boldsymbol{d}(U) * \boldsymbol{d}(V)\| = \|\boldsymbol{d}(U \triangle V)\| = \gamma(U \triangle V). \tag{1.10}$$

This implies that the element $\varepsilon(U, V)$ is dependent on the design T through the loading coefficient $\gamma(U \triangle V)$ of the loading vector $\boldsymbol{d}(U \triangle V)$ corresponding to the factorial effect $\theta\{U \triangle V\}$.

Let $\boldsymbol{\Gamma}(T)$ be a vector composed of the first row elements of the information matrix $M(T)$, i.e.,

$$\boldsymbol{\Gamma}(T) = (\gamma_\phi(T), \gamma_1(T), \ldots, \gamma_k(T), \ldots, \gamma_m(T)), \tag{1.11}$$

where $\gamma_k(T) = (\gamma\{1, 2, \ldots, k\}, \ldots, \gamma\{p_1, p_2, \ldots, p_k\}, \ldots, \gamma\{m-k+1, m-k+2, \ldots, m\})$ is the $(k+1)$st $\binom{m}{k}$ dimensional component vector of $\boldsymbol{\Gamma}(T)$. Clearly, both $\gamma_\phi(T)$ and $\gamma_m(T)$ are scalars and $\gamma_\phi(T) = \gamma(\phi) = n$. Here $\binom{n}{r}$ denotes a binomial coefficient with a usual convention.

Definition 1.3. (Yamamoto, Fujii, Hyodo and Yumiba [11,13]) A 2^m dimensional vector $\boldsymbol{\Gamma}(T)$ is called the characteristic vector of the information matrix $M(T)$ or the design T.

Using (1.10), every element of $M(T)$ can be determined completely by the characteristic vector $\boldsymbol{\Gamma}(T)$.

The first member of the normal equation $M(T)\boldsymbol{\theta} = E(T)^t \boldsymbol{y}(T)$ is given by the spur of the Schur product of the loading vector $\boldsymbol{d}(\phi)$ and the observation vector $\boldsymbol{y}(T)$, i.e.,

$$\begin{aligned} n\theta\{\phi\} + \sum_{u=1}^{m} \sum_{U \in \Omega(u)} \gamma(U)\theta\{U\} &= \|\boldsymbol{d}(\phi) * \boldsymbol{y}(T)\| \\ &= \sum_{\alpha=1}^{n} y(j_1^{(\alpha)}, j_2^{(\alpha)}, \ldots, j_m^{(\alpha)}). \end{aligned} \tag{1.12}$$

In general, the member of the normal equation in the row corresponding to $\theta\{K\}$ is given by,

$$n\theta\{K\} + \sum_{u=0}^{m} \sum_{K \neq U \in \Omega(u)} \gamma(K \triangle U)\theta\{U\} = \|\boldsymbol{d}(K) * \boldsymbol{y}(T)\|. \tag{1.13}$$

Definition 1.4. (Yamamoto, Fujii, Hyodo and Yumiba [11,13]) The linear equation (1.13) is called the principal equation for estimating the factorial effect $\theta\{K\}$.

The left hand member of the equation (1.12) has been called a defining formula.

The left hand member of (1.13) can easily be derived from that of (1.12) by multiplying $\theta\{K\}$ subject to the following symbolic operation '$\odot$' i.e.,

$$\theta\{K\} \odot \theta\{U\} = \theta\{K \triangle U\}, \tag{1.14}$$

and vice versa.

The left hand member of the equation (1.13) has also been called a derived formula.

Clearly, (1.13) provides us BLUE of the $\theta\{K\}$ if $\gamma(K \triangle U) = 0$ and/or $\theta\{U\} = 0$ by assumption for every $U \neq K$.

Definition 1.5. (Yamamoto, Fujii, Hyodo and Yumiba [11,13]) In a fractional 2^m factorial design T having the characteristic vector $\mathbf{\Gamma}(T)$, a factorial effect $\theta\{U\}$ is called

(a) orthogonal to a factorial effect $\theta\{K\}$ if $\gamma(K \triangle U) = 0$,

(b) confounded or aliased with a factorial effect $\theta\{K\}$ if $|\gamma(K \triangle U)| = n$, and,

(c) partially confounded or partially aliased with a factorial effect $\theta\{K\}$ if $0 < |\gamma(K \triangle U)| < n$.

The fraction $|\gamma(K \triangle U)|/n$ is called the confounding coefficient of $\theta\{K\}$ to $\theta\{U\}$.

Let T be a design derived from a two-symbol orthogonal array of strength 2, m constraints and index λ, i.e., 2-OA$(2, m, \lambda)$. Then, all loading coefficients $\gamma\{p\}$ of m main effects and those $\gamma\{p, q\}$ of $\binom{m}{2}$ two-factor interactions vanish simultaneously. All main effects $\theta\{p\}$ and two-factor interactions $\theta\{p, q\}$ are, therefore, orthogonal to the general mean $\theta\{\phi\}$. Moreover, every main effect can never confound with any other main effects.

§2. Orthogonal 24-run 2^6 factorial designs derivable from saturated two-symbol orthogonal arrays of size 24, strength 2 and index 6

The number of isomorphic classes of 24-run 2^6 factorial designs (or 2-OA(2,6,6)'s) derivable from saturated two-symbol orthogonal arrays of strength 2, size 24, 23 constraints and index 6, i.e., 2-OA(2,23,6)'s, isomorphic with respect to the permutation of columns (factors) and symbols (levels) within factors has been decided computationally to be 1,317 in our preceding paper (Yumiba, Hyodo and Yamamoto [15]).

The characteristics $(a) \sim (h)$ of some of those designs including significant ones are listed in Table 1.

The characteristics in Table 1 are as follows:

(a) The number ($*$) of representative orthogonal designs,

(b) The number [A $*$] of saturated orthogonal arrays which yields the design ($*$),

(c) The selected set of columns,

(d) The characteristic vector of the design,

(e) The resolution of the design,

(f) The total sum of the variances of the estimates up to two-factor interactions if the design is of resolution V,

(g) The partial sum of the variances of the estimates of main effects if the design is of resolution IV or V, and

(h) The partial sum of the variances of the estimates of two-factor interactions if the design is of resolution V.

Some of the significant aspects obtained from our results are as follows:

(i) Those 1,317 representative designs of the isomorphic classes of all orthogonal 24-run 2^6 factorial designs derivable from saturated 2-OA(2,23,6) are obtained from only 18 saturated orthogonal arrays (this number is not necessarily minimum since it depends on the order of 130 arrays treated in our program), i.e., [A1], [A2], [A11], [A12], [A13], [A14], [A15], [A16], [A30], [A31], [A32], [A34], [A35], [A37], [A38], [A48], [A71] and [A73] given in Appendix of Yamamoto, Fujii, Hyodo and Yumiba [12].

(ii) The first saturated 2-OA(2,23,6), labeled [A1], yields (1) through (41) representatives of the isomorphic classes of orthogonal 24-run 2^6 factorial designs. In addition, the second [A2] yields (42) through (208) designs, the 11th [A11] yields (209) through (359) designs, the 12th [A12] yields (360) through (945) designs, the 13th [A13] yields (946) through (1030) designs, the 14th [A14] yields (1031) through (1143) designs, the 15th [A15] yields (1144) through (1194) designs, the 16th [A16] yields (1195) through (1219) designs, the 30th [A30] yields (1220) through (1256) designs, the 31st [A31] yields (1257) through (1276) designs, the 32nd [A32] yields (1277) through (1295) designs, the 34th [A34] yields (1296) through (1301) designs, the 35th [A35] yields (1302) through (1307) designs, the 37th [A37] yields (1308) through (1310) designs, the 38th [A38] yields (1311) design, the 48th [A48] yields (1312) design, the 71st [A71] yields (1313) through (1314) designs and the 73rd [A73] yields (1315) through the last (1317) representative designs. No remaining saturated 2-OA(2,23,6) yields a new class of design.

(iii) Statistical properties of some of the representative orthogonal 24-run 2^6 factorial designs including significant ones among 1,317 representatives are given in Table 1. The loading coefficient $\gamma(K)$ in Table 1 and formula (1.10) may be used for obtaining the information matrix of each design.

(iv) Those 446 representative designs among 1,317 marked V are of resolution V, i.e., every effect up to two-factor interactions can be estimated under the assumption that three or more factor interactions are negligible.

(v) Those 60 representative designs among 1,317 marked IV are of resolution IV, i.e., all main effects can be estimated under the same assumption as (iv).

(vi) The designs (16) and (17) are orthogonal arrays of strength 3. Although the former is composed of a simple array with parameters (6;2,0,0, 1,0,0,2) (e.g., see Shirakura [5]), the latter is not composed of a balanced array. The information matrices $M(2, T(16))$ and $M(2, T(17))$ of the designs (16) and (17) can be obtained using (1.10) as follows:

The information matrices show that every main effect can be estimated independently by the corresponding principal equations. In this case, the recovery from confounding is impossible with respect to some two-factor interactions. The property of those designs is main-effect optimal under the assumption that three or more factor interactions can be neglected.

(vii) The property of the designs (739), (746) and (947) are A-optimal among the class of the resolution V orthogonal 24-run 2^6 factorial designs.

(viii) The information matrix $M(2, T(52))$ of the design (52) can be obtained using (1.10) as follows:

The information matrix shows that the main effect $\theta\{1\}$ can be estimated independently by the first principal equation. Remaining main effects are partially confounded with one or two two-factor interactions and the confounding coefficients are the same $1/3$. Those main effects, therefore, can be estimated with the same variance $(1/24)\sigma^2$ by the corresponding principal equations in as much as their confounded effects can be ignored. In this case, the elimination of confounding is possible with respect to every effect up to two-factor interactions. Comparing those two ignoring and eliminating estimates, the effect of two-factor interactions to main effects can be evaluated. This design attains minimum among the class of all resolution V orthogonal designs in as much as the partial sum of the variances of the estimates of main effects.

(ix) The information matrix $M(2, T(574))$ of the design (574) can be obtained using (1.10) as follows:

Although the confounding coefficients are the same $1/3$ and are not so large, every main effect is partially confounded with three or four or five two-factor interactions. Those main effects, therefore, can be estimated with the same variance $(1/24)\sigma^2$ by the corresponding principal equations if their confounded effects can be ignored. In this case, the elimination of confounding is possible with respect to every effect up to two-factor interactions. This design attains minimum among the class of all resolution V orthogonal designs in as much as the partial sum of the variances of the estimates of two-factor interactions.

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